Matura
podstawowowaa
7 maj 2018

Zad. 1 $2\log_3 6 - \log_3 4 = \log_3 6^2 - \log_3 4 = \log_3 \frac{36}{4} = \log_3 9$

$= \log_3 3^2 = \underline{\underline{2}}$ (B)

Zad. 2 $\sqrt[3]{\frac{7}{3}} \cdot \sqrt[3]{\frac{81}{56}} = \sqrt[3]{\frac{\cancel{7}}{\cancel{3}} \cdot \frac{\cancel{81}\,27}{\cancel{56}\,8}} = \sqrt[3]{\frac{27}{8}} = \sqrt[3]{\left(\frac{3}{2}\right)^3} = \underline{\underline{\frac{3}{2}}}$ (C)

Zad. 3 $a = 3{,}6 \cdot 10^{-12}$

$b = 2{,}4 \cdot 10^{-20}$

$\frac{a}{b} = ?$

$\frac{a}{b} = \frac{3{,}6 \cdot 10^{-12}}{2{,}4 \cdot 10^{-20}} = \frac{3{,}6}{2{,}4} \cdot 10^{-12+20} = \underline{\underline{1{,}5 \cdot 10^{8}}}$ (C)

Zad. 4 $x - 15\% \cdot x = 850$ zł

$x = ?$

$85\% \cdot x = 850$

$0{,}85x = 850 \quad /: 0{,}85$

$\underline{\underline{x = 1000 \text{ zł.}}}$

(C)

Zad. 5 $\frac{1-2x}{2} > \frac{1}{3} \quad / \cdot 6$

$3(1-2x) > 2$

$3 - 6x > 2$

$-6x > -1 \quad /: (-6)$

$x < \frac{1}{6} \implies \underline{\underline{x \in \left(-\infty; \frac{1}{6}\right)}}$ (A)

Zad. 6 $f(x) = -2(x+3)(x-5)$

$f(x_1) = f(x_2) = 0$

$x_1 + x_2 = ?$

$x_1 + x_2 = -3 + 5 = \underline{\underline{2}}$ (C)

Zad. 7 $\quad \frac{x^2+2x}{x^2-4}=0 \quad \Big| \quad x_i = \text{ile} = ?$

$\frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}=0 \quad \wedge \quad x^2-4\neq 0$

$x^2 \neq 4 \quad /\sqrt{\ }$

$|x| \neq 2$

$\frac{x}{x-2}=0 \qquad Z: \ x \neq \{-2; 2\}$

$x=0$

$\hookrightarrow$ 1 rozwiązanie

D

Zad. 8 $\quad f(x)=\frac{1}{3}x-1$

$f(x)\nearrow \quad \wedge \quad f(0)=-1 \Rightarrow P=(0,-1)\in f(x)$

D

Zad. 9 $\quad f(x)=x^2-6x-3 \quad \Big| \quad W=(p;q)=?$

$f(x)=(x-p)^2+q$

$p=\frac{-(-6)}{2\cdot 1}=3 \quad ; \quad q=f(3)=9-18-3=-12$

$W=(3;-12)$

C

Zad. 10 $\quad f(x)=ax+b \quad \Big| \quad a=?$

① $f(1)=0$

② $M=(3;-2)\in f(x)$

$$\begin{cases} a+b=0 & ① \\ 3a+b=-2 & ② \end{cases}$$

$-2a=2 \quad /:(-2)$

$a=-1$

D

Zad. 11 $\quad a_n=\frac{5-2n}{6}=-\frac{1}{3}n+\frac{5}{6} \Rightarrow$ c.a

$\underbrace{a_n=a_1+(n-1)\cdot r}_{\text{c.a}} \quad \wedge \quad r=-\frac{1}{3}$

A

Zad. 12

$a_n = a_1 + (n-1)\cdot r$ | $a_5 = ?$

① $a_4 + a_5 + a_6 = 12$

① $a_1 + 3r + a_1 + 4r + a_1 + 5r = 12$

$3a_1 + 12r = 12 \quad | : 3$

$a_1 + 4r = \underline{\underline{4 = a_5}}$

A

Zad. 13

$a_n = a_1 \cdot q^{n-1}$ | $a_n = ?$

$a_1 = \sqrt{2}$

$a_2 = 2\sqrt{2}$

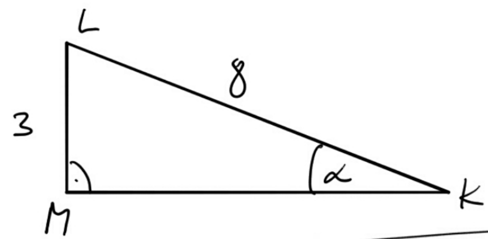
$a_3 = 4\sqrt{2}$

$\frac{a_2}{a_1} = \frac{a_1 q}{a_1} = q = 2 \Rightarrow \underline{\underline{a_n}} = \sqrt{2}\cdot 2^{n-1} = \sqrt{2}\cdot 2^n \cdot 2^{-1}$

$= \frac{\sqrt{2}\cdot 2^n}{2} = \underline{\underline{\frac{2^n}{\sqrt{2}}}}$

B

Zad. 14



$\alpha \in ?$

$\sin\alpha = \frac{3}{8} = 0,375 \xrightarrow{\text{Z TABLIC}} \alpha \approx 22^\circ$

więc $\underline{\underline{21^\circ < \alpha < 24^\circ}}$

C

Zad. 15

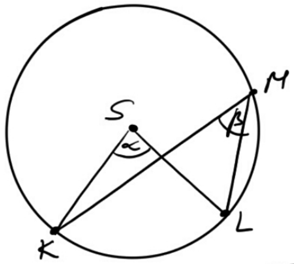
Δ_1: $a_1 = 2\sqrt{5}$, $b_1 = 3\sqrt{5}$, $c_1 = 4\sqrt{5}$ $\wedge$ $\Delta_2 \sim \Delta_1$ | $\Delta_2 = ?$

$\Delta_1: a_1 : b_1 : c_1 = 2:3:4 \Rightarrow \Delta_2: a_2 : b_2 : c_2 = \underline{10:15:20}$

$= 2:3:4$

A

Zad. 16



D:
$\alpha + \beta = 111^\circ$

Sz:
$\alpha = ?$

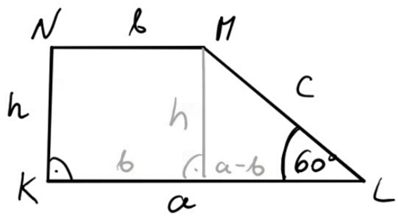
① $\overset{\frown}{KL}$: α – $\sphericalangle$ środkowy
β – $\sphericalangle$ wpisany

$\alpha = 2\beta$
$\Downarrow$
$\boxed{\beta = \frac{1}{2}\alpha}$

② $\alpha + \beta = 111^\circ$
$\alpha + \frac{1}{2}\alpha = 111^\circ$
$\frac{3}{2}\alpha = 111^\circ \quad / \cdot \frac{2}{3}$
$\alpha = 74^\circ$

A

Zad. 17

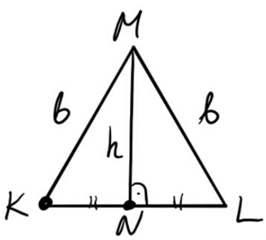


$c = ?$

① $\Delta\ 30^\circ, 60^\circ, 90^\circ \Longrightarrow c = 2(a-b)$

B

Zad. 18



$K = (2; 2)$
$N = (4; 3)$
$L = (x; y)$

$L = ?$

$\overrightarrow{KN} = \overrightarrow{NL} \Longrightarrow [2; 1] = [x-4; y-3] \Longrightarrow L = (6; 4)$

B

Zad. 19

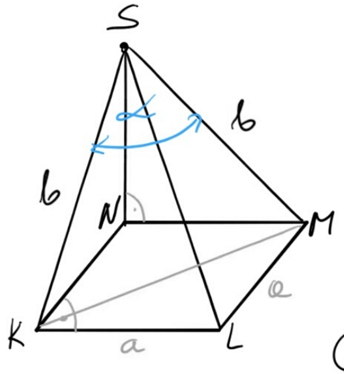
$l_1: y = (m+2)x + 3$
$l_2: y = (2m-1)x - 3$
① $l_1 \parallel l_2$

$m = ?$

① $m + 2 = 2m - 1 \Longrightarrow m = 3$

B

Zad. 20



$a=4$

$|SN|=H=4$ | $\alpha \in ?$

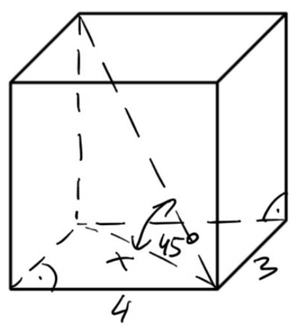
① $|KM| = a\sqrt{2} = 4\sqrt{2}$

② $b = 4\sqrt{2} \Rightarrow \Delta SNM: 45^\circ, 45^\circ, 90^\circ$

③ $\Delta KMS \Rightarrow$ równoboczny $\Rightarrow \underline{\underline{\alpha = 60^\circ}}$

D

Zad. 21



$H = ?$

TW. PITAGORASA:

① $x^2 = 3^2 + 4^2$

$x = 5$

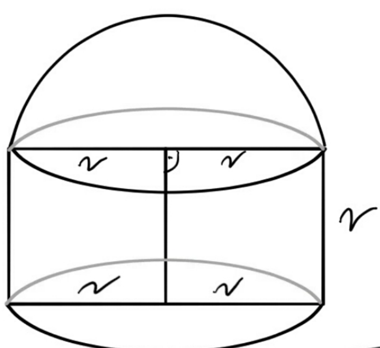
②

$\underbrace{\Delta\ 45^\circ, 45^\circ, 90^\circ}$

$\underline{\underline{H = x = 5}}$

A

Zad. 22



$V = ?$

① $P_p = \pi r^2$ → Pole podstawy (koła)

$V_1 = \pi r^2 \cdot r = \pi r^3$ → Objętość walca

$V_2 = \frac{1}{2} \cdot \frac{4}{3} \pi r^3$ → — " — dachu (kuli $\frac{1}{2}$)

② $V = V_1 + V_2 = \pi r^3 + \frac{2}{3}\pi r^3 = \underline{\underline{\frac{5}{3}\pi r^3}}$

A

Zad. 23 $Z = \{\underbrace{2;2;\dots}_{m};\underbrace{4;4;\dots}_{m}\}$

$n = 2\cdot m$

$\sigma = ?$ (odchyl. stan.)

① Średnia: $\overline{X} = \frac{2\cdot m + 4\cdot m}{2m} = \frac{6m}{2m} = 3$

② WARIANCJA:

$$\sigma^2 = \frac{(2-3)^2\cdot m + (4-3)^2\cdot m}{2m} = \frac{2m}{2m} = 1$$

③ ODCHYLENIE: $\sigma = \sqrt{1} = 1$

B

Zad. 24 $n \in N$

A - liczby naturalne 4cyfrowe podzielne przez 5 i mniejsze od 2018

$\overline{\overline{A}} = ?$

$A = 5n \quad \wedge \quad 5n \in \langle 1000; 2017 \rangle \; |:5$

$n \in \langle 200; 403{,}4 \rangle \wedge n \in N$

$n = \{200; 201; \dots; 403\}$

$|n| = 403 - 199 = 204$

$\overline{\overline{A}} = 204$

D

Zad. 25 $n = 50 = 15p + 35w$

$k = 1$

A - wyciągnięto kupon wygrywający

$P(A) = ?$

$\overline{\overline{\Omega}} = n = 50$

$\overline{\overline{A}} = 35$

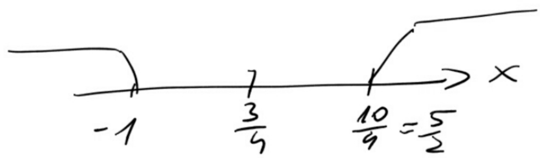
$P(A) = \frac{35}{50} = \frac{7}{10}$

D

Zad. 26 <2p>

I metoda

$$2x^2-3x>5$$
$$2x^2-3x-5>0 \quad /:2$$
$$x^2-\frac{3}{2}x-\frac{5}{2}>0$$
$$\left(x-\frac{3}{4}\right)^2-\frac{9}{16}-\frac{40}{16}>0$$
$$\left(x-\frac{3}{4}\right)^2>\frac{49}{16} \quad /\sqrt{\ }$$
$$\left|x-\frac{3}{4}\right|>\frac{7}{4}$$



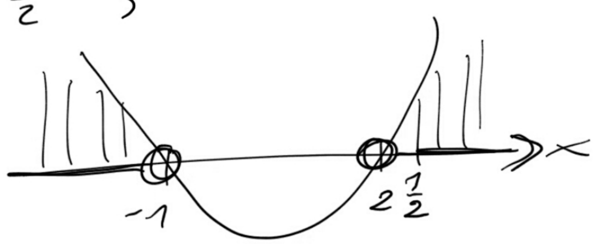
Odp: $x \in (-\infty; -1) \cup (2\frac{1}{2}; \infty)$

II metoda

np. z Δ.

$$2x^2-3x>5$$
$$2x^2-3x-5>0$$

$$\left\{ \begin{array}{l} \Delta = 49, \quad \sqrt{\Delta}=7 \\ x_1 = -1 \\ x_2 = 5/2 = 2\frac{1}{2} \end{array} \right\}$$



Odp: $x \in (-\infty; -1) \cup (2\frac{1}{2}; \infty)$

Zad. 27 <2p>

$$(x^3+125)\cdot(x^2-64)=0$$

$x^3+125=0 \quad \vee \quad x^2-64=0$

$x^3=-125 \quad /\sqrt[3]{\ } \quad \vee \quad x^2=64 \quad /\sqrt{\ }$

$\underline{x=-5}$

$|x|=8$

$\underline{x=-8} \quad \vee \quad \underline{x=8}$

odp: $\underline{\underline{x=\{-8;-5;8\}}}$

Zad. 28 <2p>

Z: $a,b>0$

T: $\frac{1}{2a}+\frac{1}{2b}\geqslant\frac{2}{a+b} \quad \Big/\cdot 2ab(a+b)>0$

D: $b(a+b)+a(a+b)\geqslant 4ab$

$ab+b^2+a^2+ab-4ab\geqslant 0$

$a^2+b^2-2ab\geqslant 0$

$(a-b)^2\geqslant 0$

$\bigwedge\limits_{a,b\in\mathbb{R}_+}(a-b)^2\geqslant 0$ $\qquad$ $\underline{\underline{\text{cnd}}}$.

II metoda: $\frac{1}{2a}+\frac{1}{2b}\geqslant\frac{2}{a+b} \quad \wedge \quad a,b>0$

$$\frac{b}{2ab}+\frac{a}{2ab}\geqslant\frac{2}{a+b}$$

$$\frac{a+b}{2ab}-\frac{2}{a+b}\geqslant 0$$

$$\frac{(a+b)^2}{2ab(a+b)}-\frac{4ab}{2ab(a+b)}\geqslant 0$$

skoro $a,b>0$ to:

$$\frac{a^2+2ab+b^2-4ab}{2ab(a+b)}\geqslant 0 \quad \Big/\cdot 2ab(a+b)>0$$

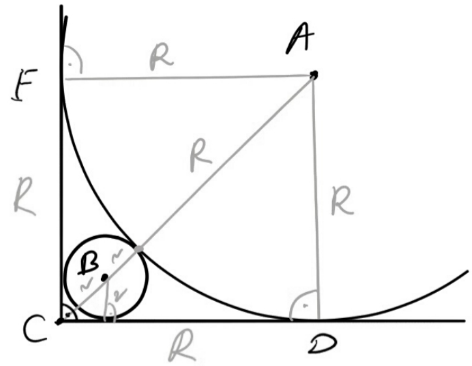
$$a^2-2ab+b^2\geqslant 0$$

$(a-b)^2\geqslant 0$ $\qquad$ $\underline{\underline{\text{cnd.}}}$

Zad. 29 <2p.>

Z: $R = 2$

T: $r \overset{?}{<} \sqrt{2} - 1$



D: $|CA|$ – przekątna □ AECD, o boku 2; więc:

$$|CA| = r\sqrt{2} + r + R \quad \wedge \quad |CA| = R\sqrt{2}$$

(1)
$$r(\sqrt{2}+1) + 2 = 2\sqrt{2}$$
$$r(\sqrt{2}+1) = 2(\sqrt{2}-1) \quad /:(\sqrt{2}+1)$$
$$r = \frac{2(\sqrt{2}-1)}{(\sqrt{2}+1)} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{2(2-2\sqrt{2}+1)}{2-1}$$
$$\underbrace{r = 2(3-2\sqrt{2}) = 6-4\sqrt{2}}$$

(2)
$$r < \sqrt{2}-1$$
$$6-4\sqrt{2} < \sqrt{2}-1$$
$$7 < 5\sqrt{2} \quad /()^2$$
$$49 < 50$$

PRAWDA $\Rightarrow r < \sqrt{2}-1$

cnd..

Zad. 30 <2p>

$f(x) = a^x$

$a > 0$

$a \neq 1$

① $P = (2; 9) \in f(x)$

② $g(x) = f(x) - 2$

$a = ?$

$ZW_g = ?$

① $f(2) = 9 \Rightarrow a^2 = 9$

$a^2 = 3^2 \Rightarrow \underline{\underline{a = 3}}$

$\underbrace{f(x) = 3^x} \wedge 3^x > 0 \Rightarrow ZW_f = (0; \infty)$

② $f(x) \xrightarrow{[0,2]} f(x) - 2 = g(x)$

$ZW_f = (0; \infty) \xrightarrow{[0,2]} \underline{\underline{ZW_g = (-2; \infty)}}$

Odp: $a = 3$; zbiór wartości funkcji $g(x)$ wynosi $(-2; \infty)$.

Zad. 31 < 2p > $n \in \mathbb{N}^+$

$a_n = a_1 + (n-1) \cdot r$

(1) $a_{12} = 30$

$S_{12} = 162$

$a_1 = ?$

(1) $\begin{cases} a_1 + 11r = 30 \\ \frac{2a_1 + 11r}{\cancel{2}} \cdot \cancel{12}^{6} = 162 \quad /:6 \end{cases}$

$-\begin{cases} a_1 + 11r = 30 \\ 2a_1 + 11r = 27 \end{cases}$

$-a_1 = 3 \quad /:(-1)$

$\underline{a_1 = -3}$

Odp: $\underline{\underline{a_1 = -3}}$

Zad. 32 <5p>

ΔABC:

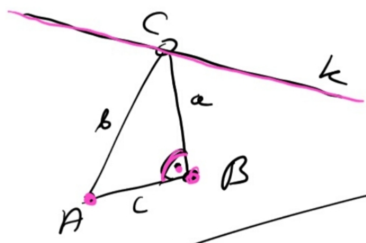
$A=(4;3)$

$B=(10;5)$

$C=(x_c;y_c)\in k$

$k:\ y=2x+3$

$|\sphericalangle ABC|=90^\circ$



$C=\ ?$

① l_{AB}: $\quad ax+b=y$

$$\begin{cases} A:\ 4a_1+b_1=3 \\ B:\ 10a_1+b_1=5 \end{cases}$$

$-6a_1=-2 \quad /:(-6) \Rightarrow \underline{a_1=\frac{1}{3}} \Rightarrow$ wsp. kier. pr. l_{AB}.

② $l_{BC}\perp l_{AB}$: $\quad \frac{1}{3}\cdot a=-1 \Rightarrow \underline{a=-3}$

$l_{BC}:\ y=-3x+b$

$B\in l_{BC}:\ 5=-3\cdot 10+b \Rightarrow b=35$

$l_{BC}:\ \underline{y=-3x+35}$

③ $C\in(l_{BC}\cap k)$:

$$\begin{cases} l_{BC}:\ y=-3x+35 \\ k:\ y=2x+3 \end{cases}$$

$2x+3=-3x+35$

$5x=32 \quad /:5$

$\underline{x=6{,}4}$

$\underline{y=2\cdot 6{,}4+3=15{,}8}$

odp: $C=(6{,}4;15{,}8)=\left(6\frac{2}{5};15\frac{4}{5}\right)$

Zad. 33 <4 p>

$A = \{100; 200; 300; 400; 500; 600; 700\}$

$B = \{10; 11; \underline{12}; 13; 14; \underline{15}; 16\}$

$k = 2$; $n = n_A + n_B = 7 + 7 = 14$

ki, BP.

M – suma wylos. liczb jest $3|$

$P(M) = ?$

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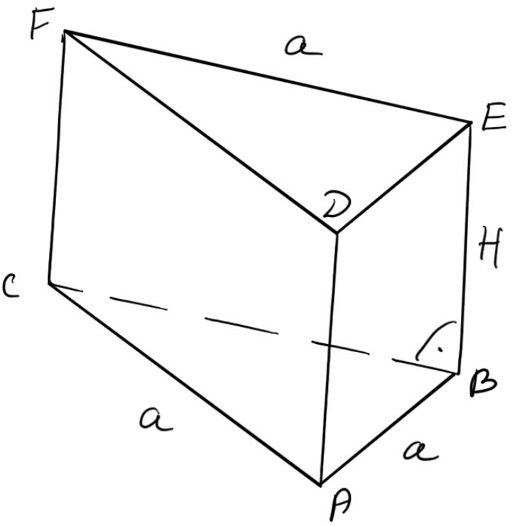
① $\overline{\overline{\Omega}} = n_A \cdot n_B = 7 \cdot 7 = 49$

② $M = \{(100; 11); (100; 14); (200; 10); (200; 13); (200; 16);$
$(300; 12); (300; 15); (400; 11); (400; 14);$
$(500; 10); (500; 13); (500; 16); (600; 12); (600; 15);$
$(700; 11); (700; 14)\}$

$\overline{\overline{M}} = 16$

③ $P(M) = \frac{16}{49}$

Zad. 34 <4p>



(2) $P_c = 45\sqrt{3}$

(1) $P_p = a \cdot H$ | $V = ?$

(1)

$$\frac{a^2\sqrt{3}}{4} = a \cdot H \quad / : a > 0$$

$$\underline{H = \frac{a\sqrt{3}}{4}}$$

(2)

$$\cancel{2} \cdot \frac{a^2\sqrt{3}}{\cancel{4}_{2}} + 3a \cdot H = 45\sqrt{3}$$

$$\frac{a^2\sqrt{3}}{2} + 3 \cdot \frac{a^2\sqrt{3}}{4} = 45\sqrt{3} \quad / \cdot \frac{8}{\sqrt{3}}$$

$$4a^2 + 6a^2 = 45 \cdot 8 \quad / : 2$$

$$2a^2 + 3a^2 = 180$$

$$5a^2 = 180 \quad / : 5$$

$$a^2 = 36 \quad / \sqrt{\ } \ \wedge \ a > 0$$

$$\underline{a = 6}$$

(3)

$$V = P_p \cdot H = \frac{a^2\sqrt{3}}{4} \cdot \frac{a\sqrt{3}}{4} = \frac{a^3 \cdot 3}{4^2} = \frac{6^3 \cdot 3}{4^2}$$

$$V = \frac{\cancel{6}^{3}{}^2 \cdot 6 \cdot 3}{\cancel{4}_{2}{}^2} = \frac{3^2 \cdot 18}{2^2} = \frac{9 \cdot \cancel{18}^{9}}{\cancel{4}_{2}} = \frac{81}{2}$$

$$\underline{V = 40{,}5} \quad [j^3].$$